

Puzzle of the Week

Maximizing Products with 16

Look at ways to break up 16 into a sum of numbers that you can then multiply to get as big a product as possible. Writing $16 = 10 + 6$ is a start, but $16 = 6 + 5 + 5$ is better. Can you do better?

$$16 = 10 + 6 \text{ and } 10 \times 6 = 60$$

$$16 = 6 + 5 + 5 \text{ and } 6 \times 5 \times 5 = 150$$

THE CHALLENGE: What is the biggest product you can make by breaking 16 into a sum of numbers?

EXPLORATION: How does your strategy change if you replace 16 with 20, 50, or 100?

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Maximizing Products with 16 – Notes

THE CHALLENGE & EXPLORATION: Let's start by looking at some examples that replace 16 with smaller numbers. Getting this experience is almost always a good way to start, and it usually doesn't take long.

$$1 = 1 \Rightarrow 1$$

$$6 = 3 + 3 \Rightarrow 3 \times 3 = 9$$

$$2 = 2 \Rightarrow 2$$

$$7 = 2 + 2 + 3 \Rightarrow 2 \times 2 \times 3 = 12$$

$$3 = 3 \Rightarrow 3$$

$$8 = 2 + 3 + 3 \Rightarrow 2 \times 3 \times 3 = 18$$

$$4 = 2 + 2 \text{ or just } 4 \Rightarrow 4$$

$$9 = 3 + 3 + 3 \Rightarrow 3 \times 3 \times 3 = 27$$

$$5 = 2 + 3 \Rightarrow 2 \times 3 = 6$$

$$10 = 2 + 2 + 3 + 3 \Rightarrow 2 \times 2 \times 3 \times 3 = 36$$

As you look at these examples, a pattern emerges for breaking down larger numbers.

1. Any number larger than 4 should be replaced with smaller numbers!
2. Never use 1 unless you have no choice.
3. Use $2 + 2$ instead of 4. They give the same result, but it is easier to see how to improve things when looking at 2's rather than 4's.
4. Always replace $2 + 2 + 2$, which gives $2 \times 2 \times 2 = 8$, with $3 + 3$, which gives $3 \times 3 = 9$.

Using these rules, a general strategy emerges for attacking any number.

1. If the number is even, write it as a sum of 2's. If the number is odd, write it as 3 plus a sum of 2's.
2. Replace any group of three 2's with a group of two 3's.

With all this in place, we are ready to solve the problem.

$$16 = 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 2 + 2 + 3 + 3 + 3 + 3, \text{ which gives a product of } 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 324.$$

As for the other numbers, we have:

$$20 = 10 \times 2 = 2 + 3 \times (3 \times 2) = 2 + 3 \times (2 \times 3) = 2 + 6 \times 3, \text{ which gives } 2 \times 3^6 = 2 \times 729 = 1458.$$

$$50 = 25 \times 2 = 2 + 8 \times (3 \times 2) = 2 + 8 \times (2 \times 3) = 2 + 16 \times 3, \text{ which gives } 2 \times 3^{16}.$$

$$100 = 50 \times 2 = 2 \times 2 \times 16 \times (3 \times 2) = 2 + 2 + 16 \times (2 \times 3) = 2 \times 2 + 32 \times 3, \text{ which gives } 4 \times 3^{32}.$$